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International Journal of Heat and Mass Transfer 43 (2000) 839–847

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

'Wall-adjacent layer' analysis for developed-flow laminar heat transfer of gases in microchannels

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Received 18 January 1999

Abstract

The wall effect on heat transfer characteristics was investigated for gas laminar flow through microchannels. In an extremely thin layer adjacent to the solid wall, we called 'wall-adjacent layer', the change in thermal conductivity of gas vs the distance from the wall surface was derived from the kinetic theory of gases. A model including the variation of thermal conductivity was built to investigate the heat transfer rule. Analytical expressions for temperature profiles and heat transfer coefficients were derived respectively for fully developed laminar flow heat transfer of gases in microtubes and in plain microchannels. The investigation shows that the change of thermal conductivity of gas in 'wall-adjacent layer' would result in significant influence on the heat transfer when passage size is small enough.

Keywords: Gas-flow convective heat transfer; Microchannels; Wall-adjacent layer; Change in thermal conductivity; Analytical model

1. Introduction

With the rapid development of fabricating microelectronics, miniaturized manufacturing and biomedical engineering etc, the microscale heating and cooling attract much more attentions in many engineering fields. It needs precise comprehension for microscale fluid flow and heat transfer.

The pioneer experiment investigations on gas flow and heat transfer in micromachined structure with plate-fin and pin-fin heat sink were reported by Tuckerman and Pease in 1984 [1]. Wu and Little [2,3] measured the friction factor for gas flow and heat transfer in the microchannels with width from 130 to 200 μm and depth 30–60 μm, the critical Reynolds number from laminar to turbulent flow was found much lower than usual, which was considered as to be caused by the large relative roughness. Choi et al. [4] studied experimentally the fluid flow and convective heat transfer in circular microtubes having dimensions from 3 to 81 μm, the friction factor was found to be less than that of the common-sized tubes and the Colburn analogy was not valid for microtubes having inside diameters less than 80 μm.

On the theoretical investigations with the consideration of time-dependent slip flow of gas and temperature jumper conditions have been investigated by Beskok and Karniadakis [5], the numerical simulation was presented for fully-developed flow between two parallel plates, inlet flows, two- and three-dimensional Couette flows, as well as slip flow past a circular cylinder. Recently, Mohiuddin Mala et al. [6] conducted

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0017-9310/00/$ - see front matter
PII: S0017-9310(99)00109-X

Nomenclature

A	constant in Eq. (1′)
a	constant defined by Eq. (19)
B	constant in Eq. (11′)
c	constant-volume specific heat
c_p	constant-pressure specific heat
d	diameter
F	heating surface area
f	friction factor
H	half width of the microchannel
h	heat transfer coefficient
J	molecular flow
k	thermal conductivity
k_b	thermal conductivity in bulk region
Kn	Knudsen number
l	mean free path
m	mass of molecule
N	number of molecules
Nu	Nusselt number
n	molecular number density
p	pressure
q	heat flux
R	radius of tube
$\bar{s}$	mean molecular speed
T	temperature
t	time
u	velocity
u_m	mean velocity
u_{max}	maximum velocity
v	volume
z_0	distance apart from the wall surface

Greek symbols

θ	dimensionless temperature
ζ	$\exp(-z_0/l)$, dimensionless
μ	dynamic viscosity
μ_b	dynamic viscosity in bulk region
τ	shear stress
η	r/R, dimensionless

theoretical investigation for laminar flow and heat transfer of liquid in metallic microchannels with the EDL (electrical double layer) theory.

In colloid and interface science, the effect of solid surface on the viscosity of fluids has attracted considerable attention. It has been shown that [7,8], the viscosity of liquids within a very thin film adjacent to the solid wall can be much different from those in bulk region. The viscosity in this near-wall layer can be higher or lower than the viscosity in bulk region, depending on the properties of the liquids. Pfahler, et al. [9] investigated the influences of the viscosity on the liquid or gas flow through microchannels with depth ranging from thousand Angstroms to tens of microns, the experimental results showed that the viscosity of the fluid adjacent to the wall surface is consistently smaller than the bulk value apart from the wall, so that the friction factor in microchannels would be smaller than that in macrochannels.

In our previous papers [10,11], we have presented a model for simulating the laminar gas flow in micropassages, based on the change of the viscosity in 'wall-adjacent layer'. This paper will report the variation of gas thermal conductivity in such a wall-adjacent layer, derived from the kinetic theory of gases. The temperature distributions and the heat transfer coefficients for laminar gas flow through a microtube or a microchannel between two parallel plates are thereby analyzed, respectively.

2. Thermal conductivity of gas flow in wall-adjacent layer

A simplified kinetic theory based on the assumption of rigid-sphere molecules was used to detect the variation of gas thermal conductivity in the wall-adjacent layer. The basic assumptions are:

1. The molecules of the gas are assumed rigid, non-attracting spheres.
2. For the size of the passages being considered, the conclusions from statistical mechanics are valuable.
3. After colliding with the surface of the wall, the gas

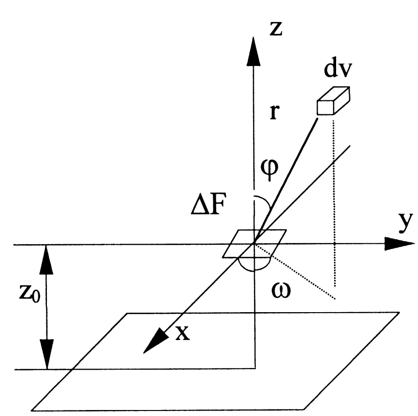

Fig. 1. Coordinate system for thermal conductivity.

molecules depart from the wall surface in all directions with same probability.

As shown in Fig. 1, there is an area element ΔF at the center of coordinate system, and a plane at the position z_0 apart from ΔF. Then, the energy exchange - taking place on ΔF would be affected by the molecular flow, dJ, from the plane surface. According to the kinetic theory of the molecules, the differential molecular flow from any volumetrical element dv to ΔF, without colliding with the other molecules, will be

$$\mathrm{d}J = \frac{n\bar{s}\cos\varphi}{4\pi l r^2}\mathrm{e}^{-r/l}\,\mathrm{d}v \tag{1}$$

or

$$\mathrm{d}J = \frac{n\bar{s}}{4\pi l}\mathrm{e}^{-r/l}\sin\varphi\cos\varphi\,\mathrm{d}\varphi\,\mathrm{d}\omega\,\mathrm{d}r. \tag{2}$$

Here, n is the number density of molecules; $\bar{s}$, the mean molecular speed; l, the mean free path of the molecules. Then, the net heat exchange taking place at ΔF would be

$$q = \int mc_v T(z)\,\mathrm{d}J \tag{3}$$

where, m is the mass of a molecule, T is temperature, and c_v is the constant-volume specific heat of the molecules. Expanding T to Taylor series, we have

$$T(z) = T_0 + T_0 r\cos\varphi + \frac{1}{2}r^2\cos^2\varphi T_0 + \cdots. \tag{4}$$

Neglecting the terms with third and higher derivatives in Eq. (4), substituting it and Eq. (2) into Eq. (3), we obtain

$$q = -\frac{mnc_\mathrm{v}\bar{s}}{4\pi l}\Bigg[\int_0^{2\pi}\mathrm{d}\omega\int_0^{\pi/2}\sin\varphi\cos\varphi\,\mathrm{d}\varphi\int_0^{\infty}\left(T_0 + r\cos\varphi T_0 + \frac{r^2}{2}\cos^2\varphi T_0\right)\mathrm{e}^{-r/l}\,\mathrm{d}r - \int_0^{2\pi}\mathrm{d}\omega\int_{\pi/2}^{\pi}\sin\omega\cos\varphi\,\mathrm{d}\varphi\int_0^{z_0/\cos\varphi}\left(T_0 + r\cos\varphi T_0 + \frac{r^2}{2}\cos^2\varphi T_0\right)\mathrm{e}^{-r/l}\,\mathrm{d}r\Bigg].$$

Noting $\rho = mn$, and wall temperature $T_w = T_0 - z_0 T_0' + z_0^2 T_0''/2$, with integration, we have

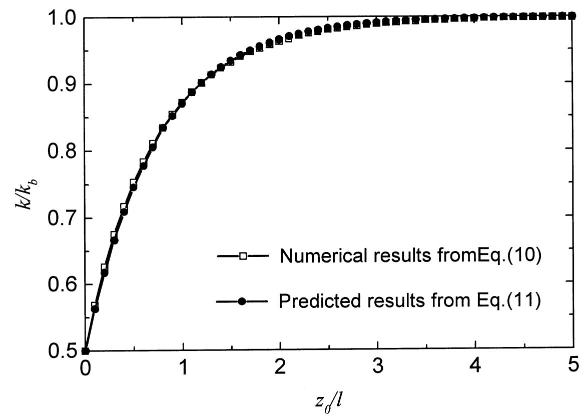

Fig. 2. Variation of gas thermal conductivity in wall-adjacent layer.

$$q = -\frac{1}{3}\rho c_\mathrm{v}\bar{s}lT_0 + \frac{\rho c_\mathrm{v}\bar{s}}{4\pi l}\int_0^{2\pi}\mathrm{d}\omega\int_{\pi/2}^{\pi}l^2 T_0\mathrm{e}^{z_0/l\cos\varphi}\sin\varphi\cos^2\varphi\,\mathrm{d}\varphi + \frac{\rho c_\mathrm{v}\bar{s}}{4\pi l}\int_0^{2\pi}\mathrm{d}\omega\int_{\pi/2}^{\pi}lT_\mathrm{w}\mathrm{e}^{z_0/l\cos\varphi}\sin\varphi\cos\omega\,\mathrm{d}\varphi. \tag{5}$$

On the right side of Eq. (5), the third term means the energy carrying by the molecular flow from the wall without colliding with the other molecules. Because of the impermeability of the solid wall, there is no such molecular flow from the wall, so that this term should be neglected. The second term can be approximated by means of the trapezoidal method as

$$\frac{\rho c_\mathrm{v}\bar{s}}{4\pi l}\int_0^{2\pi}\mathrm{d}\omega\int_{\pi/2}^{\pi}l^2T_0\mathrm{e}\int_{\pi/2}^{\pi}l^2T_0\mathrm{e}^{z_0/l\cos\varphi}\sin\varphi\cos^2\varphi\,\mathrm{d}\varphi = \frac{1}{6}\rho c_\mathrm{v}\bar{s}l\left(\zeta - \frac{z_0}{l}f(\zeta)\right) \tag{6}$$

where, $\zeta = \mathrm{e}^{-z_0/l}$, $f(\zeta) = 0.1\zeta + 0.16\zeta^{5/4} + 0.12\zeta^{5/3} + 0.08\zeta^{5/2} + 0.04\zeta^5$. Substituting Eq. (6) into Eq. (5), we have

$$q = -\frac{1}{3}\rho c_\mathrm{v}\bar{s}l\left[1 - \frac{1}{2}\left(\zeta - \frac{z_0}{l}f(\zeta)\right)\right]\frac{\partial T}{\partial z}\bigg|_{z=0}. \tag{7}$$

According to the definition, $q = -\partial T/\partial z|_{z=0}$, the thermal conductivity in the wall adjacent layer would be

$$k = \frac{1}{3}\rho c_\mathrm{v}\bar{s}l\left[1 - 0.5\zeta + 0.5\frac{z_0}{l}f(\zeta)\right]. \tag{8}$$

When $z_0 \to \infty$, $\zeta \to 0$, $f(\zeta) \to 0$, Eq. (8) takes its simpler form as

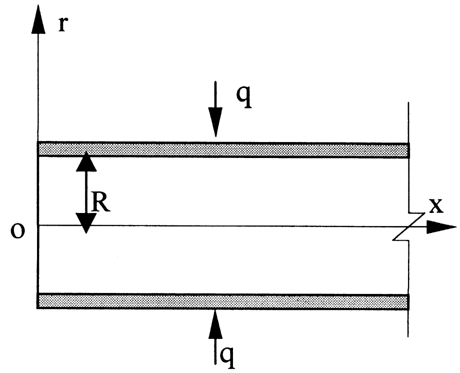

Fig. 3. Coordinate system for heat transfer of laminar gas flow in a microtube.

$$k_b = \frac{1}{3}\rho c_v \bar{s} l \tag{9}$$

k_b is just the thermal conductivity in the bulk region apart from the wall, i.e. the same as that in the case of no wall effect to be accounted [12].

Comparing Eq. (8) with Eq. (9), leads to

$$\frac{k}{k_b} = 1 - 0.5\left[\zeta - \frac{z_0}{l} f(\zeta)\right]. \tag{10}$$

We find that Eq. (10) can be correlated with good accuracy as

$$k/k_b = 1 - 0.5\mathrm{e}^{-1.35 z_0/l} \tag{11}$$

The thermal conductivity of the gas calculated from Eqs. (10) and (11) are plotted in Fig. 2, from which, k/k_b has a minimum value 0.5 on wall surface, and increases with increasing distance apart from the wall. When z_0/l equal to 5 approximately, k approaches to k_b. It is just the same as that of the viscosity variation μ/μ_b we derived in [10], i.e. the viscosity of the gas in the wall-adjacent layer taking the same form of Eq. (8) as

$$\mu = \frac{1}{3}\rho \bar{s} l\left[1 - 0.5\zeta + 0.5\frac{z_0}{l} f(\zeta)\right] \tag{12}$$

Comparing it with Eq. (8), leads to

$$k = \mu c_v. \tag{13}$$

It is similar to the relation between the viscosity and the thermal conductivity of the gas in the case of no wall effect being accounted [12].

3. Laminar-flow heat transfer in microtubes

For the case gas being in developed laminar flow, incompressible, and constant heat flux through the wall of the circular microtube, the governing equations can be built according to the coordinate system shown in Fig. 3 as

$$\begin{cases} \dfrac{1}{r}\dfrac{d}{\mathrm{d}r}\left(rk\dfrac{\mathrm{d}T}{\mathrm{d}r}\right) = \rho c_p u \dfrac{\mathrm{d}T_b}{\mathrm{d}x} \\ r = 0, \quad \dfrac{\mathrm{d}T}{\mathrm{d}r} = 0 \\ r = R, \quad T = T_w \end{cases} . \tag{14}$$

Eq. (11) can be rewritten as

$$k = k_b(1 - AE) \tag{11'}$$

where

$$E = \mathrm{e}^{-B(R-r)/l}, \quad A = 0.50, \quad B = 1.35.$$

From the energy equilibrium relation within an increment of the tube length, dx, $2\pi Rq\ \mathrm{d}x = \pi R^2 \rho c_p u_m\ \mathrm{d}T_b$, we can obtain

$$\frac{\mathrm{d}T_b}{\mathrm{d}x} = \frac{2q}{R\rho c_p u_m} \tag{15}$$

where, c_p is the specific heat of the gas, u is the velocity, u_m is the mean velocity across the section area, and T_b is the mixture-mean temperature across the section.

Let $\eta = r/R$, $\theta = (T - T_w)/(qR/k_b)$, substitute Eq. (15) into Eq. (14), a dimensionless governing equation can be built as following:

$$\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left[\eta(1 - AE)\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right] = 2\frac{u}{u_m} \tag{16}$$

with corresponding dimensionless boundary conditions

$$\eta = 0 \quad \frac{\mathrm{d}\theta}{\mathrm{d}\eta} = 0; \quad \eta = 1 \quad \theta = 0. \tag{17}$$

We have obtained the velocity distributions of developed laminar flow of the gases through the microtube as following [10]:

$$\frac{u}{u_m} = \frac{2}{1 + 8a}\left\{(1 + 4a) - \eta^2 - \frac{4}{B}Kn\eta\sum_{n=1}^{\infty}\frac{A^n}{n}E^n\right\} \tag{18}$$

where, $Kn = l/d$, is the Knudsen number, l is the mean free path, and d is the diameter of the microtube; the constant a and the function E were expressed, respectively, as

$$a = \frac{Kn}{B}\sum_{n=1}^{\infty}\frac{A^n}{n} = -\frac{Kn}{B}(1 - A) \tag{19}$$

$$E = \exp\left[-\frac{B}{2Kn}(1-\eta)\right]. \tag{20}$$

Substituting Eq. (18) into Eq. (16), and integrating it with the boundary condition (17), gives the dimensionless temperature distributions as

$$\theta = \frac{1}{1+8a}\left[(1+4a)(1-\eta^2) - \frac{1}{4}(1-\eta^4) + 2a - \frac{4Kn}{B}\left(\eta - \frac{1}{2}\eta^3\right)\sum_{n=1}^{\infty}\frac{A^n}{n}E^n\right] \tag{21}$$

Only the case for $Kn \le 0.1$, i.e. $l/d \le 0.1$ or $r/l \le 5$ were analyzed here, and so, the terms with the second and higher power of Kn in Eq. (21) could be neglected. Then, if $Kn \to 0$, Eq. (21) would be simplified to

$$\theta = \frac{3}{4} - \eta^2 + \frac{1}{4}\eta^4 \tag{22}$$

which is just the familiar result for the tubes of common size. According to the definition, the mixed temperature, T_b, would be

$$\frac{T_b - T_w}{qR/k_b} = \frac{2}{u_m}\int_0^1 \theta u \eta \, d\eta. \tag{23}$$

Substituting Eqs. (18) and (21) into Eq. (23), by integrating, gives the following equation after neglected the terms with second and higher power of the Kn:

$$\frac{T_b - T_w}{qR/k_b} = \frac{2}{(1+8a)^2}\left[\frac{11}{48} - \frac{14}{3}a + 16a^2\right]. \tag{24}$$

According to the Newton's cooling law, it is easy to derive

$$Nu = \frac{48(1+8a)^2}{11(1+224a/11+768a^2/11)} \tag{25}$$

where, the Nusselt number $Nu = hd/k_b$, and h is the heat transfer coefficient. Substituting $A = 0.5$ and $B = 1.35$ into Eq. (25), we obtain

$$Nu = \frac{4.3636(1+4.1075Kn)^2}{1+10.4556Kn+18.4057Kn^2}. \tag{26}$$

As $Kn \to 0$, Eq. (26) reduces to the conventional expression, that is, $Nu = 4.364$ for fully-developed laminar flow through a macro-sized circular tube with constant wall heat flux.

4. Heat transfer for laminar gas flow in microchannels between two parallel plates with one plate insulated thermally

Considering a coordinate system shown as Fig. 4, the heat flux through the wall $q_1 = 0$, and $q_2 = q$ = constant, for incompressible, developed laminar flow of gas, the governing equation can be established as

$$\frac{d}{d\eta}\left[(1-AE_1)\frac{d\theta}{d\eta}\right] = -\frac{u}{2u_m} \tag{27}$$

where $A = 0.5$; $\theta = (T_{w2} - T)/(qH/k_b)$ is the dimensionless temperature; u and u_m are the velocity and the mean velocity, respectively, $\eta = y/H$, with y and H shown in Fig. 4; the function E_1 was defined as $E_1 = \exp[-B(1-|\eta|)/(2Kn)]$, with $B = 1.35$ and $Kn = l/(2H)$. The corresponding boundary conditions are

$$\eta = 1, \quad \frac{d\theta}{d\eta} = 0; \quad \eta = 1, \quad \theta = 0. \tag{28}$$

The velocity distribution we derived in [11] will be

$$\frac{u}{u_m} = \frac{3}{2(1+6a)}\left[(1+4a) - \eta^2 - \frac{4Kn}{B}\mid\eta\mid\sum_{n=1}^{\infty}\frac{A^n}{n}E_1^n\right]. \tag{29}$$

Substituting Eq. (29) into Eq. (27), and solving with the boundary conditions, we have the temperature distribution as

$$\theta = \frac{1}{1+6a}\left[\left(\frac{1}{2}+3a\right)(1+\eta) + \left(\frac{3}{8}+\frac{3}{2}a\right)(1-\eta^2) - \frac{1}{16}(1-\eta^4) + 2a + \Phi(\eta)\right] \tag{30}$$

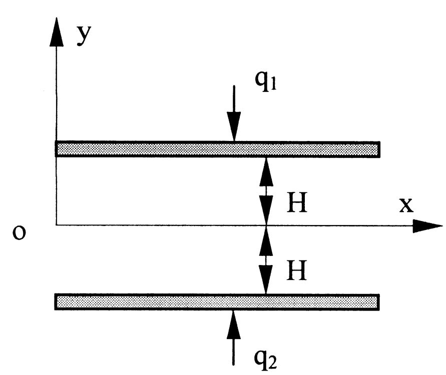

Fig. 4. Coordinate system for heat transfer of laminar gas flow in a two-dimensional microchannel.

The terms with second and higher power of Kn can be also neglected as $Kn \leq 0.1$, and the function $\Phi(\eta)$ was established as:

for $\eta \geq 0$,

$$\Phi(\eta) = \frac{Kn}{B}\left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right)\sum_{n=1}^{\infty}\frac{A^n}{n} \times \exp\left[-\frac{nB}{2Kn}(1-\eta)\right] \tag{31}$$

for $\eta < 0$,

$$\Phi(\eta) = -\frac{Kn}{B}\left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right)\sum_{n=1}^{\infty}\frac{A^n}{n} \times \exp\left[-\frac{nB}{2Kn}(1+\eta)\right]. \tag{32}$$

When $Kn \to 0$, Eq. (30) reduces to

$$\theta = \frac{1}{2}(1+\eta) + \frac{3}{8}(1-\eta^2) - \frac{1}{16}(1-\eta^4) \tag{33}$$

which is just the familiar result for developed laminar-flow heat transfer of gas through macro-sized passage between two parallel plates with one plate insulated thermally.

Based on the definition of the mixed temperature, we obtain

$$\theta_b = \frac{1}{2}\left(\int_0^1 \frac{u}{u_m}\theta\,d\eta + \int_{-1}^0 \frac{u}{u_m}\theta\,d\eta\right). \tag{34}$$

Substituting Eqs. (29) and (30) into Eq. (34), by integrating, the mixture temperature was derived as:

$$\theta_b = \frac{26}{35(1+6a)^2}\left(1 + 14a + \frac{630}{13}a^2\right). \tag{35}$$

Hence the corresponding Nusselt number for the plate 2, Nu_2 can be obtained as

$$Nu_2 = \frac{4}{\theta_b} = \frac{70(1+6a)^2}{13(1+14a+630a^2/13)} \tag{36}$$

where, the Nusselt number, $Nu_2 = 4h_2H/k_b$, and h_2 is the heat transfer coefficient of the plate 2. Because the plate 1 was heat insulated, the heat transfer coefficient is zero obviously, that is $Nu_1 = 0$. Substituting $A = 0.5$ and $B = 1.35$ into Eq. (36), Nu_2 was reduced to

$$Nu_2 = \frac{5.3846(1+3.0807Kn)^2}{1+7.1882Kn+12.7756Kn^2}. \tag{37}$$

When $Kn \to 0$, Eqs. (36) and (37) would reduce to the results from the conventional theory, or $Nu_2 = 5.3846$.

5. Heat transfer for laminar gas flow in microchannels between two parallel plates with equal constant wall heat flux

When $q_1 = q_2 = q$, because of the symmetrical temperature, a dimensionless energy equation can be set up for $\eta \geq 0$ as

$$\frac{d}{d\eta}\left[(1-AE)\frac{d\theta}{d\eta}\right] = -\frac{u}{u_m}. \tag{38}$$

The corresponding boundary conditions would be

$$\eta = 0, \quad \frac{d\theta}{d\eta} = 0; \quad \eta = 1, \quad \theta = 0 \tag{39}$$

The dimensionless temperature, $\theta = (T_w - T)/(qH/k_0)$, can be derived similarly as

$$\theta = \frac{1}{1+6a}\left[\left(\frac{3}{4} + 3a\right)(1-\eta^2) - \frac{1}{8}(1-\eta^4) + 2a - \frac{3Kn}{B}\left(\eta - \frac{1}{3}\eta^3\right)\sum_{n=1}^{\infty}\frac{A^n}{n}E^n\right] \tag{40}$$

where, $A = 0.5$, $B = 1.35$; and the constant a take the same form like Eq. (19), except $Kn = l/(2H)$.The dimensionless mixed temperature would be

$$\theta_b = \int_0^1 \frac{u}{u_m}\theta\,d\eta. \tag{41}$$

Substituting Eqs. (29) and (40) into Eq. (41) and integrating, we can obtain

$$\theta_b = \frac{17}{35(1+6a)^2}\left(1 + 14a + \frac{840}{17}a^2\right). \tag{42}$$

The Nusselt number for either plate, $Nu_1 = Nu_2 = Nu$, would be

$$Nu = \frac{4}{\theta_b} = \frac{140(1+6a)^2}{17(1+14a+840a^2/17)}. \tag{43}$$

Since $A = 0.5$ and $B = 1.35$, Eq. (43) would be

$$Nu = \frac{8.2353(1+3.0807Kn)^2}{1+7.1882Kn+13.0260Kn^2}. \tag{44}$$

When $Kn \to 0$ Eqs. (43) and (44) would reduce to $Nu = 8.2353$, which is just the same value for conventional-sized passage.

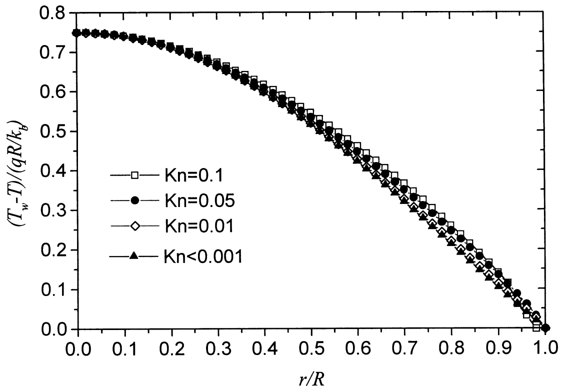

Fig. 5. Temperature distributions for laminar flow of gas in microtubes.

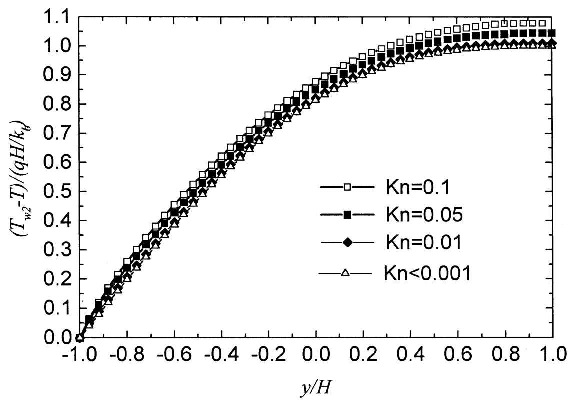

Fig. 7. Temperature distributions for laminar flow of gas in two-dimensional plain microchannels with one plate surface insulated thermally.

6. Results and discussion

Figs. 5 and 6 show the dimensionless temperature profiles and the Nusselts number from Eqs. (21) and (26) for gas laminar flow in circular microtubes, respectively.

Figs. 7 and 8 are the dimensionless temperature profiles calculated from Eqs. (30) and (40) for gas laminar flow through microchannels between two parallel plates, while Figs. 9 and 10 show the Nusselts number calculated from Eqs. (37) and (44), respectively. These calculating results show the extraordinary heat transfer characteristics for developed laminar flow of gas under microscale conditions.

The temperature profiles of microtubes (Fig. 5) and of the plain microchannels (Fig. 8) indicate that the dimensionless temperature increases with decreasing passage size, or with increasing *Kn*. The temperature profile in microtube (Fig. 5) has two common values, the same as that in macrotube: one being zero at tube center; and the other one being nearly a constant value at the tube wall surface. A little difference may be caused by the curvature of the tube wall surface. The temperature profile in plain microchannels has only one common value, at the wall surface.

In macroscale conditions, *Nu* will be constant for developed laminar flow of gas both in a circular tube or between two-parallel plates, respectively. In microscale conditions, *Nu* depends on the size of the passages, the 'wall -adjacent layer' affects the flow and heat transfer, such that *Nu* decreases with decreasing passage size, or decreases with increasing *Kn*.

The heat transfer analysis from 'wall-adjacent layer' model is comparative to those from slip-flow and jump temperature model. For example, the results shown in Fig. 6 have the similar tendency with that presented by

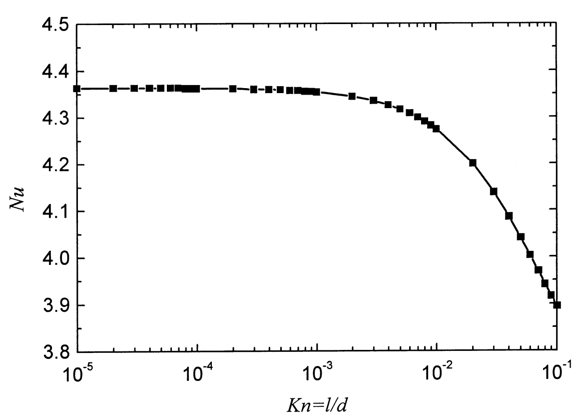

Fig. 6. Heat transfer for laminar flow of gas in microtubes.

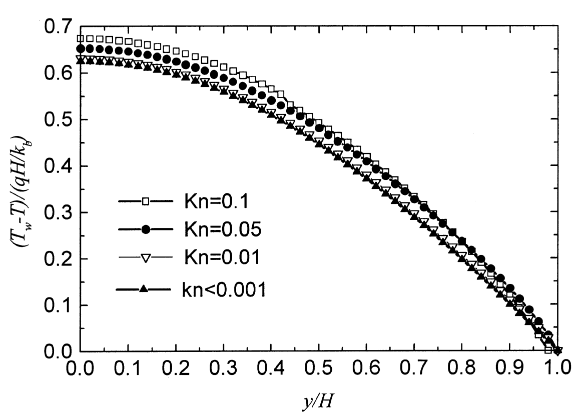

Fig. 8. Temperature distributions for laminar flow of gas in two-dimensional plain microchannels.

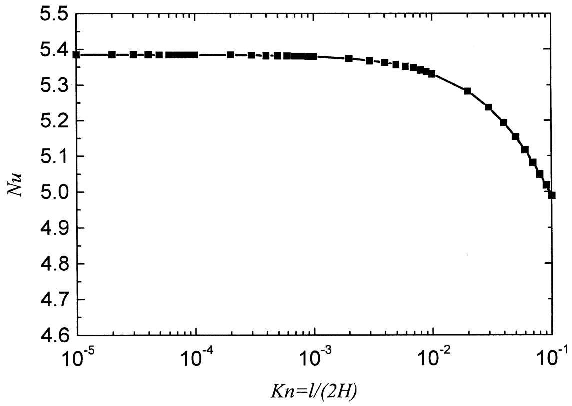

Fig. 9. Heat transfer for laminar flow of gas in two-dimensional plain microchannels with one plate surface insulated thermally.

Sparrow and Lin [13] for gas laminar heat transfer in tubes under slip-flow condition, but the value for a defined Kn is larger slightly than that predicted by Sparrow and Lin [13]. The important difference between those two models may lie in the difference of the temperature and velocity profiles. According to the slip-flow model, a sharp variation of both velocity and temperature exist in so-called slip layer, while those from 'wall-adjacent layer' model these profile are smooth, without such a sharp-variable region.

The present calculation show that, the error caused by taking the passages as conventional macroscale ones for gas laminar flow and heat transfer less than 0.1% when $Kn = 10^{-4}$, and increases to 1.0% when $Kn = 10^{-3}$, then to about 9% if Kn = 0.1. So, the critical Knudsen number from macrochannels to microchannels may be taken as 10^{-3}.

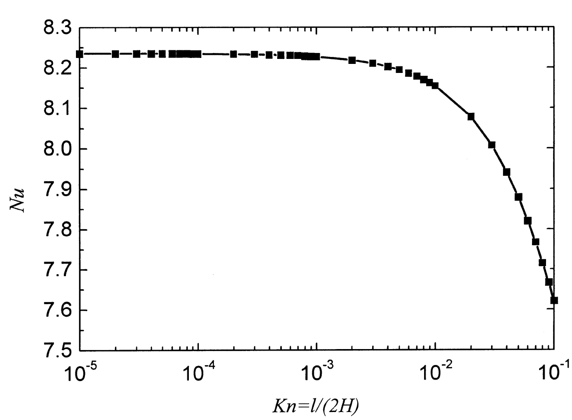

Fig. 10. Heat transfer in two-dimensional plain microchannels, with constant equal heat flux, $q_1 = q_2 = q$.

7. Conclusions

The wall-adjacent layer effect on convective heat transfer for developed laminar flow of gas in micropassages was analyzed theoretically. In a extremely tiny region adjacent to the surface of passage wall, we called as 'wall-adjacent layer', the transport properties of the gases would differ from those in bulk region and vary with the distance apart from the wall surface. Because this effect could not be neglected, the gas laminar flow and heat transfer in micropassages are different from that in macropassages.

The thickness of the wall-adjacent layer is about 3 to 5 times of the mean free path of gas molecules. If the thickness of the wall-adjacent layer could not be neglected comparing with the size of the passage, this size effect should be taken into consideration. According to this analysis, the criterion from microscale to macroscale may be taken as $Kn = 10^{-3}$.

Acknowledgements

The research project is financially supported by National Natural Science Foundation of China with grant number 9598025.

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